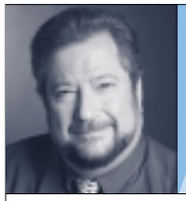


Aaron Brown

On Betting the Right Size

Many investment managers focus on the quality of individual bets and not their sizing ...

I took my title from a wonderful short essay, *On Being the Right Size*, written by the British geneticist J.B.S. Haldane in 1928. It opens: "The most obvious differences between different animals are differences of size, but for some reason the zoologists have paid singularly little attention to them. In a large textbook of zoology before me I find no indication that the eagle is larger than the sparrow, or the hippopotamus bigger than the hare, though some grudging admissions are made in the case of the mouse and the whale. But yet it is easy to show that a hare could not be as large as a hippopotamus, or a whale as small as a herring. For every type of animal there is a most convenient size, and a large change in size inevitably carries with it a change of form."

Many traders and investment managers practice a similar neglect, focusing on the quality of individual bets and paying insufficient attention to their sizing. "Bet" in this context can mean anything from a single trade or position up to a firm's entire book or portfolio. Relative sizing decisions must be made at each level. Things are easiest to discuss at the strategy level.

"I heard there was a really BIG mattress around here ..."

Strategic thinking

A strategy begins with an investment idea. Nonpractitioners are often shocked at how simple ideas are, and how few ideas there are relative to the number of investment managers. One old popular idea is to borrow money in currencies with low interest rates, swap the currencies in the spot market for currencies with high interest rates, and put the money to work in high-interest deposits or short-term fixed-income securities. Long-term testing shows that this idea works on average, although you have occasional big losses when a high interest rate currency weakens against the low interest rate currency.

Different managers will implement this idea differently, and success depends on the details. Do you define "high" and "low" interest rates in absolute terms, or relative to projected inflation rates, or in some other way? Do you try to pick pairs of countries with similar economies to reduce risk? What types of borrowing and investing do you do, and at what maturities? Do you make an effort to avoid currencies with significant risk of sharp weakening?

A quant will gather all the data she can: historical currency exchange rates and interest

rates, macroeconomic data, central bank actions, and anything else that might be related to the idea. Statistical quants will analyze empirical relationships, engineering quants will build models. Both of them will come up with computer programs that involve hundreds of positions in dozens of currencies, with continuous trading (not necessarily a large amount of trading, but continuous adjustments based on updated data from feeds). The statistician will be able to tell you why the model likes one currency or another; the engineer is likely to have a black box model with mysterious output that must be taken on faith.

A qual will read and think a lot instead. He will focus in on a few currencies and learn all about them, including studying things that seem far removed from exchange and interest rates. He will tend to trade less frequently but more dramatically, sometimes taking a position to zero suddenly or even reversing it. He hopes to have a better risk/reward ratio on each bet he makes than a quant owing to his superior research and expertise; the quant hopes to have a better portfolio risk/reward ratio through diversification.

If you only have one idea, relative size isn't an issue (absolute size is still important for many strategies, as is dynamic sizing over time). But a trader or trading desk allocating a risk budget to several ideas, or a pension fund dividing investment funds among different managers or a multistrategy hedge fund, all have to decide how to size each strategy.

Portfolio thinking

The classic portfolio management solution is simple. Construct the covariance matrix of your strategies and their expected return vector (minus financing costs, so that each one is an excess expected return). Multiply the inverse covariance matrix by the expected return vector and you get the vector of strategy weights that maximizes the Sharpe ratio.

An equivalent rule to size strategy A is to regress its returns on all the other strategies in the portfolio. Multiply the betas by the expected returns of each strategy; this gives the expected return of a proxy portfolio of the other strategies with the highest correlation to strategy A. Subtract the proxy return from the expected return of strategy A, and then divide by the variance of the residual from the regression. This gives you the optimal size for strategy A.

Of course, not everyone believes in maximizing Sharpe ratios or sizing by least squares regression. But the general rule makes sense anyway. Figure out how much strategy A adds to the risk of the rest of the portfolio, and penalize it for this fraction of the expected return of the rest of the portfolio. After all, to keep risk constant when you add strategy A, you have to reduce the size of everything else, so you have to subtract that cost from the expected return of strategy A (strategy A could, of course, reduce the risk of the rest of the portfolio if it has negative correlation, in which case you would add to its expected return). Divide the residual return of strategy A by some measure of its risk to decide its size. Even quals go through some kind of thought process analogous to this.

It bothers some people that the risk (variance) in this rule is in units of dollars-squared. One way to think about this is that variance adds across strategies to give total variance, while standard deviation does not. Another is to suppose you switched units from dollars to pennies. In this case, you would want the sizes to shrink by a factor of 100 to leave the answer unchanged; therefore, the denominator has to be in the square of the numerator units.

Sharpe or size?

An interesting question is: which is more important, individual strategy Sharpe ratio or the sizing decision? Consider two multistrategy hedge funds. One spends all its research energy on finding strategies with good Sharpe ratios. It sizes strategies in proportion to their attractiveness (this is the theoretical optimum if the strategies are independent). The other fund dedicates some attention to estimating correlations among strategies and adjusting sizes accordingly. How much can the second fund afford to give up in individual strategy Sharpe ratio to have the same overall Sharpe ratio?

For two strategies with individual Sharpe ratios of S_1 and S_2, and correlation ρ, the formula below gives the ratio of the portfolio Sharpe ratio or the optimally sized portfolio to one with strategies sized in proportion to the individual Sharpe ratio:

$$\sqrt{1-\frac{\left(2\rho S_1 S_2\right)^2}{\left(S_1^2+S_2^2\right)^2}}\Big/\sqrt{1-\rho^2}$$

For example, if S_1 is 0.5 and S_2 is 0.7, with ρ = 0.25, the formula gives 1.0035, meaning that you increase the portfolio Sharpe ratio by 0.35 percent (from 0.7736 to 0.7763) by using optimal sizing ($13 in strategy one and $23 in strategy two) instead of sizing proportionate to individual Sharpe ratio ($15 in strategy one and $21 in strategy two).

An interesting question is: which is more important, individual strategy Sharpe ratio or the sizing decision?

In practice, you rarely see two-strategy improvements more than 5 percent because people don't run strategies with vastly different Sharpe ratios and high correlations. If you find a situation like this, you combine them into one strategy. Typical values for pairwise improvement are 0 percent to 4 percent.

On the other hand, this improvement is available for every pair of strategies. The squared improvements are additive, so the total improvement goes up linearly with the number of strategies. For 10 or 20 strategies, the total improvement can add up to significant levels. Unfortunately, estimating the optimal strategy becomes increasingly difficult as the number of dimensions increases.

Table 1. Sharpe Ratios by Sector

Sector	Sharpe
Convertible arbitrage	0.78
Dedicated short bias	(0.22)
Emerging markets	0.41
Equity market neutral	1.97
Event driven	1.25
Distressed	1.34
Multistrategy	1.05
Risk arbitrage	0.90
Fixed income arbitrage	0.39
Global macro	0.94
Long/short equity	0.77
Managed futures	0.30

Sharpe or cov?

My guess is that 25 percent is about the practical limit for improvements due to correlation analysis. For an empirical example, I downloaded the CS/Tremont Hedge Fund Sector indices from 1994 to 2008. Table 1 shows the Sharpe ratios by sector.

If you had weighted every strategy equally, your portfolio Sharpe ratio would have been 4.62.

Suppose, in 1993, someone had offered to tell you the actual Sharpe ratios of the strategies from 1994 to 2008, or the actual covariance matrix over the same period. Which would have done more to improve your risk-adjusted return? I assume for this exercise that you can go long or short any of the indices in any amount.

Knowing the actual Sharpe ratios would allow you to short dedicated short bias and load up on the best strategies like equity market neutral, event-driven, and distressed. Your Sharpe ratio would have improved from 4.62 (equal weighting) to 5.59, assuming you weighted strategies in proportion to the Sharpe ratio. Knowing the full covariance matrix, but nothing at all about expected returns, would give you a Sharpe ratio of 6.35. So, it's significantly more important to know correlations among strategies than individual strategy stand-alone attractiveness.

Table 2 shows the weights selected by the investor who knew the actual Sharpe ratio, the investor who knew the actual covariance matrix, and the investor who knew both (in which case the Sharpe ratio would have been 8.47). The negative return of the short bias strategy fooled the Sharpe ratio investor into shorting it. Its attractive negative correlations with other strategies led the covariance investor to overweight it. The investor who knew both things gave it an average weight.

The other big difference is the event-driven sector. This had the third-highest Sharpe ratio, so the Sharpe ratio investor overweighted it. It also had high correlations with some other strategies, so the covariance investor made it a large short. The investor who knew both made it an even bigger short, because it happened to have high correlations with the two strategies that had higher average returns over the period.

I'm a risk manager, and we take a completely different approach. For one thing, we don't believe in covariance matrices and we laugh at inverse ones

Table 2. Weights Selected

Sector	Sharpe (percent)	Covariance (percent)	Both (percent)
Convertible arbitrage	8	(3)	(2)
Dedicated short bias	(2)	13	7
Emerging markets	4	0	(3)
Equity market neutral	19	10	10
Event driven	12	(22)	(29)
Distressed	13	15	23
Multistrategy	10	9	17
Risk arbitrage	9	9	3
Fixed income arbitrage	4	8	(1)
Global macro	9	(4)	2
Long/short equity	7	4	1
Managed futures	3	2	(0)

No suicide

I'm a risk manager, and we take a completely different approach. For one thing, we don't believe in covariance matrices and we laugh at inverse ones (the joke is that the inverse depends on the inverse eigenvalues, and there are always some eigenvalues so close to zero that they determine the result, and the data give no useful information about the smallest eigenvalues; even if the process worked statistically, it would fail numerically, as rounding errors corrupt the result). We don't think anyone knows anything about expected return. Standard deviation is kiddy-risk, like roller coasters. It gives the MBAs a thrill. The physicists love to compute stuff about it, like maximum jerk and centripetal force, after simplifying the problem beyond recognition to make it tractable. But no one dies on the roller coaster; people die on the much-less-exciting highways and slipping in bathtubs.

The risk manager's rule is simple: no suicide. Size each strategy so that it won't kill you. If you can survive the bad strategies, you can collect the gains from the good ones.

One aspect of the no suicide rule is to think each strategy out completely beforehand. What will you do if it turns against you? Stop losses? Double up? Recalibrate? Hold on? It's not enough to look only at the downside; you also need to know how you plan to realize profits. Having no exit plan after success has caused as many disasters as having no fallback plan after failure.

It's important to remember that the strategy determines the risk, not the security. A volatile security can be held in a safe way, and a low-volatility security can be held in a dangerous way.

As Haldane explained: "...suppose that a gazelle, a graceful little creature with long thin legs, is to become large, it will break its bones unless it does one of two things. It may make its legs short and thick, like the rhinoceros, so that every pound of weight has still about the same area of bone to support it. Or it can compress its body and stretch out its legs obliquely to gain

stability, like the giraffe. I mention these two beasts because they happen to belong to the same order as the gazelle, and both are quite successful mechanically, being remarkably fast runners."

If you want to make your idea larger, you have to modify the strategy.

The results of this analysis give you some idea of how much you can lose if markets are normal.

Next, you need to consider what happens if:

- Markets move faster or more than anyone thought possible;
- Illiquidity prevents you from executing your strategy;
- Changes in the terms, availability, or cost of leverage force you to unwind your strategy;
- External factors, such as redemptions or portfolio rules, require size reductions;
- Other entities in the same or similar strategies liquidate suddenly;
- Strategy-specific events, such as natural disasters, legal changes, failure of a market participant, derail your strategy;
- Things you thought had to move together, don't;
- Model errors or operational failures mean that the strategy is not what it was supposed to be.

Size is critical in these considerations; again, from Haldane:

"You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, a horse splashes."

But don't think that smaller is always safer. As Haldane says:

"But there is a force which is as formidable to an insect as gravitation to a mammal. This is surface tension. A man coming out of a bath carries with him a film of water about one-fiftieth of an inch in thickness. This weighs roughly a pound. A wet mouse has to carry about its own weight of water. A wet fly has to lift many times its own weight and, as everyone knows, a fly once wetted by water or any other liquid is in a very serious position indeed."

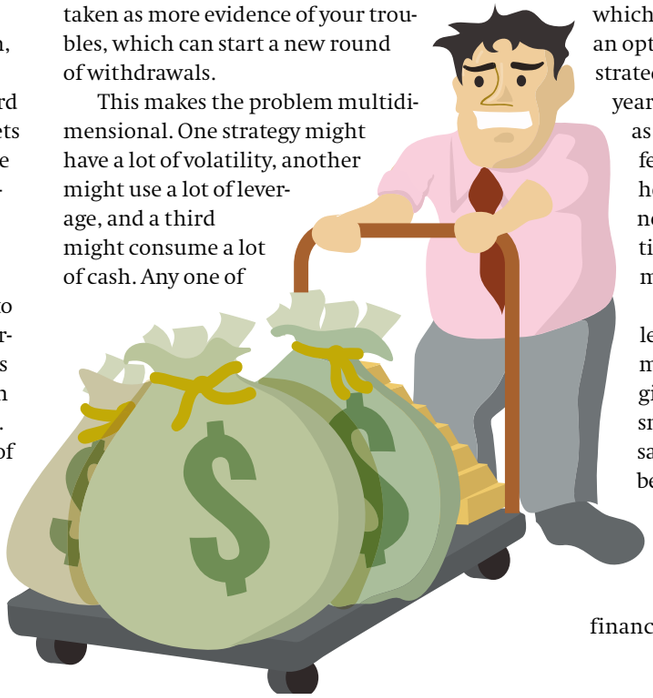
In most cases, the size constraint is not set by the result of total disaster in one strategy, but by a death spiral. A market event causes problems in some strategies, and causes people to deliver unrelated but more liquid strategies. This causes illiquidity, tight leverage, investor nervousness, and credit fears. It could be marketwide, or specific to your portfolio niche.

If both your debt and equity capital are locked in on fixed terms, all you have to know is that the combined bad-case strategy losses will very likely be less than your equity. But if your leverage can be reduced or your equity withdrawn, you have more to worry about. If A suspects that B might believe that C fears you will not survive, then A, B, and C may all try to be the first to withdraw their capital. Both the withdrawals directly and the trading losses they can force will be taken as more evidence of your troubles, which can start a new round of withdrawals.

This makes the problem multidimensional. One strategy might have a lot of volatility, another might use a lot of leverage, and a third might consume a lot of cash. Any one of these three things might be the binding constraint on portfolio survival probability. Each of them aggregates differently among strategies, and strategies can offset each other's risks instead of adding to them.

As a result, strategy sizing must be tailored to your capital strengths and weaknesses. It should be directional – that is, you are more concerned about positions that will suffer losses in times of financial stress than the opposite positions. It must consider the perceptions of your capital providers, in addition to financial reality. It should consider the order in which strategies are likely to be stressed in a bad market.

Despite that evidence, many people neglect the natural size of strategies. They believe that it is always safer to have smaller strategies and more of them



Haldane wrote:

"Such are a very few of the considerations which show that for every type of animal there is an optimum size. Yet although Galileo demonstrated the contrary more than three hundred years ago, people still believe that if a flea were as large as a man it could jump a thousand feet into the air. As a matter of fact the height to which an animal can jump is more nearly independent of its size than proportional to it. A flea can jump about two feet, a man about five."

The past 25 years demonstrate a similar lesson in finance. Despite that evidence, many people neglect the natural size of strategies. They believe that it is always safer to have smaller strategies and more of them. In fact, safety is more nearly independent of the number of strategies than proportional to it (or to its square root). Some believe in *ad hoc* sizing rules, others use quantitative techniques that miss most of the risk.

To survive, in life you must be, in finance you must bet, the right size.